

Z' boson mixings with Z - γ and charge assignments

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Z' boson mixings with $Z-\gamma$ and charge assignments

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ABSTRACT: Based on the general description for $Z'-Z-\gamma$ mixing as derived from the electroweak chiral Lagrangian, we characterize and classify the various new physics models involving the Z' boson that have appeared in the literature into five classes: 1. Models with minimal $Z'-Z$ mass mixing; 2. Models with minimal $Z'-Z$ kinetic mixing; 3. Models with general $Z'-Z$ mixing; 4. Models with $Z'-\gamma$ kinetic and $Z'-Z$ mixing; and 5. Models with Stueckelberg-type mixing. The corresponding mixing matrices are explicitly evaluated for each of these classes. We constrain and classify the Z' boson charges with respect to quark-leptons by anomaly cancellation conditions.

KEYWORDS: Spontaneous Symmetry Breaking, Beyond Standard Model, Chiral Lagrangians

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1 Introduction

With the running of the LHC at CERN Geneva, a TeV energy era begins and researchers are anxiously expecting a possible new revolution in particle physics. There are various predictions from both the Standard Model (SM) and new physics models. Among these the appearance of possible new underlying interactions beyond conventional strong/weak/electromagnetic gauge interactions is of special interest. From knowledge accumulated in recent years in particle physics, we know that the expected new interactions at least must govern the electroweak symmetry breaking that result in the massive W^\pm and Z^0 bosons and may further be responsible for the origin of masses for ordinary quarks and leptons. Theoreticians have also touted various ambitious alternative sources of these new interactions, such as unifications, supersymmetries, and extra dimensions. With the exception of the well-known scalar-type interactions which suffer unnaturalness, triviality and hierarchy problems, the typical new interaction that avoids the shortcomings of elementary scalar fields is a gauge interaction and minimal such kind of interaction involves an additional so-called $U(1)'$ gauge interaction. In most instances this extra $U(1)'$ gauge force is a "relic" of some larger underlying new physics gauge interactions such as those occurring in GUT models, string theories, left-right symmetric models and models deconstructed from extra space dimensions. Alternatively, in some special models, the $U(1)'$ gauge force takes on a special role: for example 1) in little Higgs type models, it can partially remove the quadratic divergence from the SM Higgs mass at the one loop level [1]; 2) in topcolor-assisted technicolor (TC2) models, it ensures top quark condensation while not for the bottom quark [2–4]; 3) in SUSY models, it can mediate SUSY breaking [5]; and 4) in models based on string theory, it mediates particles communicating between the hidden and visible sectors [6]. This represents but a sampling of new physics models involving additional $U(1)'$ factors: a recent review of others can be found in ref. [7].

Phenomenologically, we are interested in the possibility of experimentally finding the carrier, an electrically-neutral color singlet spin-one boson Z' , of this additional gauge force especially at the LHC. As a detection has not been made so far, this boson has to be massive and the corresponding $U(1)'$ gauge symmetry must be violated. The more preferred and exciting experimental finding would be that the Z' mass is relatively light compared with the other new physics particles, for then it might arise as a first signature of the new physics beyond SM at the LHC. This prospect heightens the need for theoretical studies of such a light Z' boson and its interactions with known particles would also be of the special importance in new physics research.

Physically, one main effect of the Z' boson derives from its mixings with conventional Z boson and γ photon; another stems from its gauge couplings to ordinary quarks and leptons, which leads to various charge assignments. There exist a diversity of new physics models involving the Z' boson, each model has its own arrangement of $Z' - Z - \gamma$ mixings and Z' coupling to ordinary quarks and leptons. To compare models, a model independent investigation is needed of these Z' boson interactions with known particles, particularly in classifying and comparing the role of the Z' boson within each model. The electroweak chiral Lagrangian (EWCL) method provides such a platform to perform model independent research. In our previous paper [8], we have written down the bosonic part up to order p^4 of the most general EWCL involving the Z' boson¹ and known particles. This EWCL also describes the most general $Z' - Z - \gamma$ mixings, and with it we can further classify the various $Z' - Z - \gamma$ mixings that appear in each model enabling us to compare and discriminate between the different new physics models.² Here the classification categorizes the general $Z' - Z - \gamma$ mixings into several simplifying cases that appear in the new physics models in the literature. The reason in doing this is because the general $Z' - Z - \gamma$ mixings is too complex to be discussed analytically, while we will show that for all simplifying cases presented in this paper, mixings can be diagonalized exactly. This improves on the approximate diagonalization result usually used in the literature and we can exhibit explicitly the relationship between the various simplifying cases. The main purpose of this paper is to present these findings and moreover to generalize the EWCL given in ref. [8] to include the Z' boson couplings to ordinary quarks and leptons for the most general charge assignments. In terms of these charges, new physics models involving the Z' boson can also be classified. Because most of the experimental searches for the Z' boson depend heavily on these charge assignments and on how Z' mixes with Z and γ , we combine a discussions on these two issues in present paper.

This paper is organized as follows. In section 2, we first give a short review of the bosonic part of the EWCL involving the Z' boson and general $Z' - Z - \gamma$ mixings. In section 3, we classify the various models involving the Z' boson that have appear in the literatures according to their arrangements of the $Z' - Z - \gamma$ mixings. In section 4, we set

¹In the Lagrangian, terms involving a neutral Higgs boson that only plays a passive role are also included to help in matching unitarity requirements within the theory.

²It should be emphasized that a p^4 order EWCL provides some special degrees of freedom for the $Z' - Z - \gamma$ mixings. For example, all kinetic mixings are from p^4 order terms in EWCL (see eq. (2.9)), as a p^2 order EWCL only cannot offer the most general $Z' - Z - \gamma$ mixings.

up the general Z' boson charge assignments to the ordinary quarks and leptons in terms of the anomaly cancellation conditions. Section 5 provides a summary of the paper.

2 The bosonic part of the EWCL involving the Z' boson and $Z' - Z - \gamma$ mixings

As given in ref. [8], the covariant derivative in the EWCL including the Z' boson is

$$D_\mu \hat{U} = \partial_\mu \hat{U} + igW_\mu \hat{U} - i\hat{U} \frac{\tau_3}{2} g' B_\mu - i\hat{U} (\tilde{g}' B_\mu + g'' X_\mu) I, \quad (2.1)$$

where the two by two unitary field \hat{U} represents four Goldstone boson degrees of freedom resulting from spontaneous symmetry breaking of $SU(2)_L \otimes U(1)_Y \otimes U(1)' \rightarrow U(1)_{\text{em}}$, and \tilde{g}' is a Stueckelberg-type coupling constant associated with which is a special kind of $U(1)$. To help in understanding this choice of covariant derivative, we denote $SU(2)_L \otimes U(1)_Y \otimes U(1)'$ group elements as $(e^{i\theta^a t_L^a + i\theta' t'}, e^{i\theta t})$ for which the Hermitian matrices t_L^a (θ^a) with $a = 1, 2, 3$, t (θ) and t' (θ') are generators (group parameters) of $SU(2)_L$, $U(1)_Y$ and an extra $U(1)'$ respectively. The electromagnetic $U(1)_{\text{em}}$ group generator has now been generalized from its traditional expression to $t_{\text{em}} \equiv t_L^3 + t + ct'$ depending on an additional arbitrary parameter c . This generator results in the $U(1)_{\text{em}}$ group element $(e^{i\theta_{\text{em}}(t_L^3 + ct')}, e^{i\theta_{\text{em}} t})$ and we can label the representative element for the corresponding coset by $(\hat{U}, 1)$. Group theory tells us that each symmetry breaking generator corresponds to a coset which can be represented by introducing a representative element for each coset. Denoting the representative element by n , its transformation rule to n' under the action of an arbitrary group element g is then $gn = n'h$ where h is an element belonging to the un-broken subgroup. Specifically for the above gauge group, this transformation rule then stipulates that

$$(e^{i\theta^a t_L^a + i\theta' t'}, e^{i\theta t})(\hat{U}, 1) \stackrel{gn=n'h}{=} \underbrace{(e^{i\theta^a t_L^a + i\theta' t'} \hat{U} e^{-i\theta(t_L^3 + ct')}, 1)}_{\hat{U}'} \underbrace{(e^{i\theta(t_L^3 + ct')}, e^{i\theta t})}_{U(1)_{\text{em}}} \quad (2.2)$$

which yields the following transformation rule for the Goldstone field \hat{U} under $SU(2)_L \otimes U(1) \otimes U(1)'$

$$\hat{U}' = e^{i\theta^a t_L^a + i\theta' t'} \hat{U} e^{-i\theta(t_L^3 + ct')} . \quad (2.3)$$

The choice of the Goldstone field in the two dimensional internal space corresponds in taking the generator $t_L^a = \tau^a/2$, $t = t' = 1$ (Note, according to our arrangement of group elements, t and t' act on different spaces, so $t = t' = 1$ will not cause confusion). With (2.3) and the standard $SU(2)_L \otimes U(1)_Y \otimes U(1)'$ transformation rule for electroweak gauge fields W_μ, B_μ and the extra $U(1)'$ gauge field X_μ , we derive the action of the covariant derivative on the Goldstone field \hat{U} as: $D_\mu \hat{U} = \partial_\mu \hat{U} + i(gW_\mu + g_X X_\mu) \hat{U} - i\hat{U} (\frac{\tau_3}{2} g' + cg') B_\mu$. Further identifying $g_X \equiv -g''$ and $cg' \equiv \tilde{g}'$, we obtain the result given in eq. (2.1). With symmetry breaking pattern $SU(2)_L \otimes U(1)_Y \otimes U(1)' \rightarrow U(1)_{\text{em}}$, the Higgs mechanism ensures that the Goldstone bosons represented by the \hat{U} field will be eaten out by the electroweak gauge bosons W^\pm, Z^0 and Z' which then acquire mass. Here W_μ, B_μ and X_μ are respectively the gauge fields of $SU(2)_L, U(1)_Y$ and $U(1)'$ before mixing.

The full bosonic part of the Lagrangian up to order p^4 is

$$\mathcal{L}_{Stueck-SU(2)_L \otimes U(1)_Y \otimes U(1)' \rightarrow U(1)_{em}} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4, \quad (2.4)$$

with each term in the Lagrangian defined as

$$\mathcal{L}_0 = -V(h), \quad (2.5)$$

$$\begin{aligned} \mathcal{L}_2 = & \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{4}f^2 \text{tr}[\hat{V}_\mu \hat{V}^\mu] + \frac{1}{4}\beta_1 f^2 \text{tr}[T\hat{V}_\mu] \text{tr}[T\hat{V}^\mu] + \frac{1}{4}\beta_2 f^2 \text{tr}[\hat{V}_\mu] \text{tr}[T\hat{V}^\mu] \\ & + \frac{1}{4}\beta_3 f^2 \text{tr}[\hat{V}_\mu] \text{tr}[\hat{V}^\mu] + \beta_4 f(\partial^\mu h) \text{tr}[\hat{V}_\mu], \end{aligned} \quad (2.6)$$

$$\mathcal{L}_4 = \mathcal{L}_K + \mathcal{L}_B + \mathcal{L}_H + \mathcal{L}_A, \quad (2.7)$$

where $T \equiv \hat{U}\tau_3\hat{U}^\dagger$ and $\hat{V}_\mu \equiv (\hat{D}_\mu\hat{U})\hat{U}^\dagger$. Here the Higgs field h is treated as p^0 order and

$$\begin{aligned} \mathcal{L}_K = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\text{tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} \\ \mathcal{L}_B = & \frac{1}{2}\alpha_1 gg' B_{\mu\nu} \text{tr}[TW^{\mu\nu}] + \frac{i}{2}\alpha_2 g' B_{\mu\nu} \text{tr}[T[\hat{V}^\mu, \hat{V}^\nu]] + i\alpha_3 g \text{tr}[W^{\mu\nu}[\hat{V}^\mu, \hat{V}^\nu]] + \dots \\ \mathcal{L}_H = & (\partial_\mu h) \left\{ \alpha_{H,1} \text{tr}[T\hat{V}^\mu] \text{tr}[\hat{V}_\nu \hat{V}^\nu] \right. \\ & \left. + \alpha_{H,2} \text{tr}[T\hat{V}_\nu] \text{tr}[\hat{V}^\mu \hat{V}^\nu] + \alpha_{H,3} \text{tr}[T\hat{V}_\nu] \text{tr}[T[\hat{V}^\mu, \hat{V}^\nu]] + \dots \right\}. \end{aligned}$$

All coefficients in above Lagrangian are functions of Higgs field h . Detailed expressions can be found in ref. [8].

Mixings among $Z' - Z - \gamma$ come from the gauge boson mass term \mathcal{L}_M and kinetic term \mathcal{L}_K . In the unitary gauge $\hat{U} = 1$, they become

$$\begin{aligned} \mathcal{L}_{M,Z'-Z-\gamma} = & \frac{f^2}{8}(1-2\beta_1)(gW_\mu^3 - g'B_\mu)^2 + \frac{f^2}{2}(1-2\beta_3)(g''X_\mu + \tilde{g}'B_\mu)^2 \\ & + \frac{f^2}{2}\beta_2(g''X_\mu + \tilde{g}'B_\mu)(gW^{3,\mu} - g'B^\mu), \end{aligned} \quad (2.8)$$

$$\begin{aligned} \mathcal{L}_{K,Z'-Z-\gamma} = & -\frac{1}{4}B_{\mu\nu}B_{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}(1-\alpha_8 g^2)(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3)^2 \\ & + \frac{1}{2}\alpha_1 gg' B_{\mu\nu}(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3) + gg''\alpha_{24}X^{\mu\nu}(\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3) \\ & + g'g''\alpha_{25}B_{\mu\nu}X^{\mu\nu}. \end{aligned} \quad (2.9)$$

Apart from the four gauge couplings g, g', g'', \tilde{g}' , seven extra dimensionless parameters $\beta_1, \beta_2, \beta_3$ and $\alpha_1, \alpha_8, \alpha_{24}, \alpha_{25}$ determine the mixing terms. Of these eleven, α_8 can be absorbed into the redefinition of field W_μ^3 and coupling constant g by

$$W_\mu^3 \rightarrow \frac{W_\mu^3}{\sqrt{1-\alpha_8 g^2}} \quad g \rightarrow g\sqrt{1-\alpha_8 g^2}. \quad (2.10)$$

Hence we are left with ten parameters, and on eliminating the three gauge couplings g, g', g'' , leaves us *seven independent* parameters $\tilde{g}', \beta_1, \beta_2, \beta_3, \alpha_1, \alpha_{24}, \alpha_{25}$ that are related to mixings. However, the mixing masses and kinetic terms given by (2.8) and (2.9) are so complex that to diagonalize them we must exploit the general 3×3 rotation matrix U_{ij}

$$(W_\mu^3, B_\mu, X_\mu)^T = U(Z_\mu, A_\mu, Z'_\mu)^T, \quad (2.11)$$

which has nine matrix elements. The fact that no correction terms arise for the kinetic terms $-\frac{1}{4}B_{\mu\nu}B_{\mu\nu}$ and $-\frac{1}{4}X_{\mu\nu}X^{\mu\nu}$ leads to two constraints on the matrix elements of U ,

$$(U^{-1,T}U^{-1})_{22} = (U^{-1,T}U^{-1})_{33} = 1, \quad (2.12)$$

which imply that there are only seven independent matrix elements. This is consistent with the earlier result that there are at most seven parameters $\tilde{g}', \beta_1, \beta_2, \beta_3, \alpha_1, \alpha_{24}, \alpha_{25}$ related to mixings. In ref. [8], we had obtained a set of relations between matrix elements U_{ij} and parameters $g, g', g'', \tilde{g}', \beta_1, \beta_2, \beta_3, \alpha_1, \alpha_8, \alpha_{24}, \alpha_{25}$ as follows

$$U \equiv \begin{pmatrix} \frac{1}{2g}c_\alpha & \frac{1}{2g} & -\frac{1}{2g}s_\alpha \\ -\frac{1}{2g'}c_\alpha & \frac{1}{2g'} & \frac{1}{2g'}s_\alpha \\ \frac{1}{g''}(s_\alpha + \frac{\tilde{g}'}{2g'}c_\alpha) & -\frac{\tilde{g}'}{2g'g'} & \frac{1}{g''}(c_\alpha - \frac{\tilde{g}'}{2g'}s_\alpha) \end{pmatrix} \begin{pmatrix} \frac{c_\beta}{A_1} & 0 & \frac{s_\beta}{A_1} \\ ga & gb & gc \\ -\frac{s_\beta}{A_2} & 0 & \frac{c_\beta}{A_2} \end{pmatrix} \begin{pmatrix} \frac{M_Z}{f} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{M_{Z'}}{f} \end{pmatrix}, \quad (2.13)$$

where $c_\alpha \equiv \cos \alpha_{Z'}$, $s_\alpha \equiv \sin \alpha_{Z'}$, $s_\beta = \sin \beta_{Z'}$, $c_\beta = \cos \beta_{Z'}$ as well as the following definitions

$$\begin{aligned} A_1^2 &= \frac{1}{4}(1-2\beta_1)c_\alpha^2 + \beta_2s_\alpha c_\alpha + (1-2\beta_3)s_\alpha^2 \\ A_2^2 &= \frac{1}{4}(1-2\beta_1)s_\alpha^2 - \beta_2s_\alpha c_\alpha + (1-2\beta_3)c_\alpha^2, \end{aligned} \quad (2.14)$$

$$\begin{aligned} \tan \alpha_{Z'} &= \frac{3 + 2\beta_1 - 8\beta_3 - \sqrt{(3 + 2\beta_1 - 8\beta_3)^2 + 16\beta_2^2}}{4\beta_2} \\ \tan \beta_{Z'} &= \frac{-G_2 + \sqrt{G_2^2 + 4G_0^2}}{2G_0} \end{aligned} \quad (2.15)$$

$$\begin{aligned} a &= \frac{1}{gA_1A_2[g'^2g''^2 - g^2g'^2g''^2(2\alpha_1 + \alpha_8) + g^2g''^2 - 4g^2g'g''^2\tilde{g}'(\alpha_{24} + \alpha_{25}) + g^2\tilde{g}'^2]} \\ &\quad \times \left\{ [g^2g''^2 + g^2\tilde{g}'^2 - g'^2g''^2 + g^2g'^2g''^2\alpha_8 + 4g^2g'g''^2\tilde{g}'\alpha_{25}](s_\alpha s_\beta A_1 + c_\alpha c_\beta A_2) \right. \\ &\quad \left. + [2g^2g'\tilde{g}' + 4g^2g'^2g''^2(\alpha_{24} + \alpha_{25})](-c_\alpha s_\beta A_1 + s_\alpha c_\beta A_2) \right\}. \\ b^2 &= \frac{4g'^2g''^2}{(g^2 + g'^2)g''^2 + g^2\tilde{g}'^2 - g^2g'^2g''^2(2\alpha_1 + \alpha_8) + 4g^2g'g''^2\tilde{g}'(\alpha_{24} + \alpha_{25})}. \\ c &= \frac{1}{gA_1A_2[g'^2g''^2 - g^2g'^2g''^2(2\alpha_1 + \alpha_8) + g^2g''^2 - 4g^2g'g''^2\tilde{g}'(\alpha_{24} + \alpha_{25}) + g^2\tilde{g}'^2]} \\ &\quad \times \left\{ [g^2g''^2 + g^2\tilde{g}'^2 - g'^2g''^2 + g^2g'^2g''^2\alpha_8 + 4g^2g'g''^2\tilde{g}'\alpha_{25}](-s_\alpha c_\beta A_1 + c_\alpha s_\beta A_2) \right. \\ &\quad \left. + [2g^2g'\tilde{g}' + 4g^2g'^2g''^2(\alpha_{24} + \alpha_{25})](c_\alpha c_\beta A_1 + s_\alpha s_\beta A_2) \right\}. \end{aligned}$$

$$\begin{aligned}
 G_0 = & -A_1 A_2 \left\{ (-g^2 - g'^2 + g''^2 + (\tilde{g}')^2) c_\alpha s_\alpha + g' \tilde{g}' (s_\alpha^2 - c_\alpha^2) + g^2 [2g'^2 c_\alpha s_\alpha + g' \tilde{g}' (c_\alpha^2 - s_\alpha^2)] \alpha_1 \right. \\
 & + g^2 [(g'^2 - g''^2 - (\tilde{g}')^2) c_\alpha s_\alpha - g' \tilde{g}' (s_\alpha^2 - c_\alpha^2)] \alpha_8 + 2g^2 g''^2 (c_\alpha^2 - s_\alpha^2) (\alpha_{24} + g'^2 \alpha_1 \alpha_{25}) \\
 & + g''^2 [-4g' \tilde{g}' c_\alpha s_\alpha + 2g'^2 (c_\alpha^2 - s_\alpha^2)] [g^2 (\alpha_8 \alpha_{25} - \alpha_1 \alpha_{24}) - \alpha_{25}] + g^2 g''^2 [8g'^2 s_\alpha c_\alpha \\
 & + 4g' \tilde{g}' (c_\alpha^2 - s_\alpha^2)] \alpha_{24} \alpha_{25} + g^2 g'^2 g''^2 s_\alpha c_\alpha (4\alpha_{25}^2 - \alpha_1^2) \\
 & \left. + 4g^2 g''^2 (g' s_\alpha + \tilde{g}' c_\alpha) (g' c_\alpha - \tilde{g}' s_\alpha) \alpha_{24}^2 \right\} \\
 G_2 = & A_1^2 \left\{ (g^2 + g'^2) c_\alpha^2 + (g''^2 + (\tilde{g}')^2) s_\alpha^2 (1 - g^2 \alpha_8) - g^2 g'^2 c_\alpha^2 (2\alpha_1 + \alpha_8) \right. \\
 & + 4g' g''^2 \tilde{g}' s_\alpha^2 \alpha_{25} - 4g^2 g'^2 g''^2 c_\alpha^2 (\alpha_{24}^2 + \alpha_{25}^2 + 2\alpha_{24} \alpha_{25}) \\
 & \left. - g^2 g''^2 s_\alpha^2 [g'^2 \alpha_1^2 + 4(\tilde{g}')^2 \alpha_{24}^2 + 4g' \tilde{g}' (\alpha_8 \alpha_{25} - \alpha_1 \alpha_{24})] \right\} \\
 & - [A_1 \rightarrow A_2, c_\alpha \leftrightarrow s_\alpha] + s_\alpha c_\alpha (A_1^2 + A_2^2) \left\{ -2g' \tilde{g}' [1 - g^2 (\alpha_1 + \alpha_8)] \right. \\
 & \left. + 4g^2 g''^2 [(\alpha_{24} - \alpha_{25}) (1 - g''^2 \alpha_1) + 2g' \tilde{g}' \alpha_{24}^2 + g''^2 \alpha_8 \alpha_{25}] \right\}. \quad (2.16)
 \end{aligned}$$

Finally the masses of Z and Z' bosons are determined from

$$\mathbf{K}_{11} = -\frac{1}{4} \quad \mathbf{K}_{33} = -\frac{1}{4}, \quad (2.17)$$

with

$$\mathbf{K} \equiv U^T \begin{pmatrix} -\frac{1}{4}(1 - \alpha_8 g^2) & \frac{1}{4} \alpha_1 g g' & \frac{1}{2} g g'' \alpha_{24} \\ \frac{1}{4} \alpha_1 g g' & -\frac{1}{4} & \frac{1}{2} g' g'' \alpha_{25} \\ \frac{1}{2} g g'' \alpha_{24} & \frac{1}{2} g' g'' \alpha_{25} & -\frac{1}{4} \end{pmatrix} U. \quad (2.18)$$

General expressions for the mixing matrix elements U_{ij} are too complicated to be written analytically. In ref. [8], we listed results for U_{ij} , M_Z and $M_{Z'}$ expanded up to order p^4 and linear in \tilde{g}' . In real new physics models appearing in the literature, the $Z' - Z - \gamma$ mixings are often not so complex. In the next section, we identify and discuss typical $Z' - Z - \gamma$ mixings appearing in various new physics models.

3 Classification of models in terms of their $Z' - Z - \gamma$ mixings

In this section, we organize the various new physics models that can be found in the literature involving the Z' boson according to their $Z' - Z - \gamma$ mixings. Unlike the most general case reviewed in the last section, these mixings are special $Z' - Z - \gamma$ mixings for which the mixing matrix elements U_{ij} and M_Z , $M_{Z'}$ can all be work out exactly. Below we consider five situations.

1. Minimal $Z' - Z$ mass mixing [1, 9–21]:

This kind of model provides minimal mixing by ignoring all mixings in the kinetic terms and $Z - \gamma$, $Z' - \gamma$ mixings in the mass terms. They correspond to the vanishing five parameters

$$\tilde{g}' = \alpha_1 = \alpha_8 = \alpha_{24} = \alpha_{25} = 0. \quad (3.1)$$

With the exception of gauge couplings g, g', g'' , the remaining three nontrivial parameters are denoted by the $Z' - Z$ mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} M_{Z_0}^2 & M_{ZZ'}^2 \\ M_{ZZ'}^2 & M_{Z'_0}^2 \end{pmatrix} \quad Z_0^\mu \equiv \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \quad A_0^\mu \equiv \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}} \quad Z_0'^\mu \equiv X^\mu, \quad (3.2)$$

where mass parameters $M_{Z_0}^2$, $M_{Z'_0}^2$ and $M_{ZZ'}^2$ are related to $\beta_1, \beta_2, \beta_3$ as

$$\frac{f^2}{4}(1-2\beta_1)(g^2+g'^2) \equiv M_{Z_0}^2 \quad f^2(1-2\beta_3)g''^2 \equiv M_{Z'_0}^2 \quad \frac{f^2}{2}\beta_2 g'' \sqrt{g^2+g'^2} \equiv M_{ZZ'}^2. \quad (3.3)$$

refs. [19, 20] use an alternative expression which corresponds to setting

$$f = v_H \quad g' = g_Y \quad g'' = g_z \quad \beta_1 = 0 \quad \beta_2 = -\frac{1}{2}z_H \quad 1 - 2\beta_3 = \frac{1}{4}\left(z_H^2 + \frac{v_\phi^2}{f^2}\right)$$

Ref. [21] further generalizes this which leads then to

$$g' = g_Y \quad g'' = g_z \quad 1 - 2\beta_1 = \frac{v_{H_1}^2 + v_{H_2}^2}{f^2} \quad \beta_2 = -\frac{z_{H_1}v_{H_1}^2 + z_{H_2}v_{H_2}^2}{2f^2} \\ 1 - 2\beta_3 = \frac{1}{4f^2}(z_{H_1}^2 v_{H_1}^2 + z_{H_2}^2 v_{H_2}^2 + v_\phi^2).$$

In this kind of model, the key $Z' - Z$ mixing parameter is β_2 which yields a non-vanishing off-diagonal element $M_{ZZ'}^2$ in the $Z' - Z$ mass matrix. This element further generates the seesaw splitting between the original Z and Z' masses,

$$M_Z^2 = \frac{1}{2}\left[M_{Z_0}^2 + M_{Z'_0}^2 - \sqrt{(M_{Z_0}^2 - M_{Z'_0}^2)^2 + 4M_{ZZ'}^4}\right] \approx M_{Z_0}^2 - \frac{M_{ZZ'}^4}{M_{Z'_0}^2 - M_{Z_0}^2} \quad (3.4)$$

$$M_{Z'}^2 = \frac{1}{2}\left[M_{Z_0}^2 + M_{Z'_0}^2 + \sqrt{(M_{Z_0}^2 - M_{Z'_0}^2)^2 + 4M_{ZZ'}^4}\right] \approx M_{Z'_0}^2 + \frac{M_{ZZ'}^4}{M_{Z'_0}^2 - M_{Z_0}^2}. \quad (3.5)$$

Meanwhile the $Z' - Z$ mixing can be parameterized by mixing angle θ'

$$\begin{pmatrix} Z_0^\mu \\ Z_0'^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta' & \sin \theta' \\ -\sin \theta' & \cos \theta' \end{pmatrix} \begin{pmatrix} Z^\mu \\ Z'^\mu \end{pmatrix} \quad \tan 2\theta' = \frac{2M_{ZZ'}^2}{M_{Z'_0}^2 - M_{Z_0}^2}. \quad (3.6)$$

leading to a rotation matrix introduced in (2.11) of the form

$$U_{\text{Minimal } Z'-Z \text{ mass mixing}} = \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\sin \theta_W & \cos \theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta' & 0 & \sin \theta' \\ 0 & 1 & 0 \\ -\sin \theta' & 0 & \cos \theta' \end{pmatrix} \\ = \begin{pmatrix} \cos \theta_W \cos \theta' & \sin \theta_W \cos \theta' & \cos \theta_W \sin \theta' \\ -\sin \theta_W \cos \theta' & \cos \theta_W \cos \theta' & -\sin \theta_W \sin \theta' \\ -\sin \theta' & 0 & \cos \theta' \end{pmatrix}, \quad (3.7)$$

with an electroweak mixing angle $\tan \theta_W = g'/g$.

2. Minimal $Z' - Z$ kinetic mixing [22–24]:

This kind of model provides minimal mixing by ignoring all mixings in the mass terms and $Z - \gamma$, $Z' - \gamma$ mixings in the kinetic terms leading to the vanishing of seven parameters

$$\tilde{g}' = \beta_1 = \beta_2 = \beta_3 = \alpha_1 = \alpha_8 = \alpha_{24} = 0. \quad (3.8)$$

Again with the exception of gauge couplings g, g', g'' , the one remaining nontrivial parameter is denoted by

$$g' g'' \alpha_{25} \equiv -\frac{\sin \chi}{2}. \quad (3.9)$$

following ref. [22], we redefine the gauge fields as

$$B^\mu = B_0^\mu - \tan \chi Z_0^{\prime\mu} \quad X^\mu = \frac{Z_0^{\prime\mu}}{\cos \chi}, \quad (3.10)$$

in terms of the fields $B_0^\mu, Z_0^{\prime\mu}, W^{3\mu}$, the kinetic term appears diagonalized and the model reduces to a minimal $Z' - Z$ mass mixing model discussed above³ with

$$M_{Z_0}^2 = \frac{f^2}{4}(g^2 + g'^2) \quad M_{Z_0'}^2 = \frac{f^2[g'^2 \sin^2 \chi + 4g''^2]}{4 \cos^2 \chi} \quad M_{ZZ'}^2 = \frac{f^2}{4} g' \sqrt{g^2 + g'^2} \tan \chi. \quad (3.11)$$

The rotation matrix introduced in (2.11) takes the form

$$\begin{aligned} & U_{\text{Minimal } Z' - Z \text{ kinetic mixing}} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\tan \chi \\ 0 & 0 & \frac{1}{\cos \chi} \end{pmatrix} \times U_{\text{Minimal } Z' - Z \text{ mass mixing}} \\ &= \begin{pmatrix} \cos \theta' \cos \theta_W & \sin \theta_W & \cos \theta_W \sin \theta' \\ -\sin \theta_W \cos \theta' + \tan \chi \sin \theta' & \cos \theta_W & -\sin \theta_W \sin \theta' - \tan \chi \cos \theta' \\ -\sin \theta' / \cos \chi & 0 & \cos \theta' / \cos \chi \end{pmatrix}. \end{aligned} \quad (3.12)$$

3. General $Z' - Z$ mixing [1–4, 6, 25–27]:

This kind of model is combinations of minimal $Z' - Z$ mass mixing model and minimal $Z' - Z$ kinetic mixing model discussed above which correspond to

$$\tilde{g}' = \alpha_1 = \alpha_8 = \alpha_{24} = 0 \quad g' g'' \alpha_{25} \equiv -\frac{\sin \chi}{2}. \quad (3.13)$$

In a similar manner as for minimal $Z' - Z$ kinetic mixing model, we can use (3.10) to remove the mixing in the kinetic term and then, in terms of the fields $B_0^\mu, Z_0^{\prime\mu}, W^{3\mu}$, the model can be changed into a minimal $Z' - Z$ mass mixing model with identifications

$$\begin{aligned} M_{Z_0}^2 &= \frac{f^2}{4}(1 - 2\beta_1)(g^2 + g'^2) \\ M_{Z_0'}^2 &= \frac{f^2[g'^2(1 - 2\beta_1) \sin^2 \chi + 4g''^2(1 - 2\beta_3) + 4\beta_2 g' g'' \sin \chi]}{4 \cos^2 \chi} \\ M_{ZZ'}^2 &= \frac{f^2(1 - 2\beta_1)g' \sin \chi + 2\beta_2 g'' \sqrt{g^2 + g'^2}}{4 \cos \chi}. \end{aligned} \quad (3.14)$$

³This detail was not pointed out in ref. [22].

The resulting rotation matrix has the same form as in (3.12), the only change is that now the θ' as determined through (3.6) is different due to the new expressions for $M_{Z_0}^2, M_{Z'_0}^2, M_{Z Z'}^2$ given by (3.14). In some dynamical models such as TC2 models, the general $Z' - Z$ mixings are generated by technicolor and topcolor dynamics, as in refs. [28–30], while mixing parameters are given through dynamical computations depending on the nature of the TC2 models and results in the following expressions

$$g' g'' \alpha_{25} = \frac{g'^2 \gamma}{2c_{Z'}} \quad \frac{f^2}{2} \beta_2 g'' = \frac{g'}{4c_{Z'}} \times \begin{cases} (F_0^{\text{TC2}})^2 \tan \theta' & \text{Ref. [2, 28]} \\ 3(F_0^{1\text{D}})^2 \tan \theta' & \text{Ref. [3, 29]} \\ -3(F_0^{1\text{D}})^2 \cot \theta' & \text{Ref. [4, 30]} \end{cases}, \quad (3.15)$$

where all symbols appearing on the right-hand side of these results are parameters pertaining to the TC2 models.

4. $Z' - \gamma$ kinetic and $Z' - Z$ mixing [31]:

B. Holdom extends the conventional $Z' - Z$ mixing by further adding in model a $Z' - \gamma$ kinetic mixing term. His model corresponds to having

$$\begin{aligned} \tilde{g}' = \alpha_1 = \alpha_8 = 0 & \quad \frac{f^2}{4}(1 - 2\beta_1) = m_Z^2 \\ \frac{f^2}{2} \beta_2 g'' \sqrt{g^2 + g'^2} = x m_Z^2 & \quad f^2(1 - 2\beta_3) g''^2 = m_X^2 \\ g g'' \sqrt{g^2 + g'^2} \alpha_{24} = -\frac{1}{2}(g y + g' w) & \quad g g'' \sqrt{g^2 + g'^2} \alpha_{25} = \frac{1}{2}(g' y - g w). \end{aligned} \quad (3.16)$$

We can diagonalize the kinetic terms by redefining the B^μ and $W^{3\mu}$ fields as

$$\begin{aligned} B^\mu &= B_0^\mu - \frac{\sin \chi}{\sqrt{1 - \sin^2 \chi - \sin^2 \bar{\chi}}} Z_0'^\mu & W^{3\mu} &= W_0^{3\mu} - \frac{\sin \bar{\chi}}{\sqrt{1 - \sin^2 \chi - \sin^2 \bar{\chi}}} Z_0'^\mu \\ X^\mu &= \frac{Z_0'^\mu}{\sqrt{1 - \sin^2 \chi - \sin^2 \bar{\chi}}} & -\frac{\sin \bar{\chi}}{2} &\equiv g' g'' \alpha_{24} & -\frac{\sin \chi}{2} &\equiv g' g'' \alpha_{25} \end{aligned} \quad (3.17)$$

and then in terms of fields $B_0^\mu, Z_0'^\mu, W_0^{3\mu}$, the model becomes the minimal $Z' - Z$ mass mixing model with

$$\begin{aligned} M_{Z_0}^2 &= \frac{f^2}{4}(1 - 2\beta_1)(g^2 + g'^2) \\ M_{Z'_0}^2 &= \frac{f^2[\frac{1}{4}(1 - 2\beta_1)(g' \sin \chi - g \sin \bar{\chi})^2 + (1 - 2\beta_3)g''^2 + \beta_2 g''(g' \sin \chi - g \sin \bar{\chi})]}{1 - \sin^2 \chi - \sin^2 \bar{\chi}} \\ M_{Z Z'}^2 &= \frac{f^2 [(1 - 2\beta_1)(g' \sin \chi - g \sin \bar{\chi}) + 2\beta_2 g'']}{4 \sqrt{1 - \sin^2 \chi - \sin^2 \bar{\chi}}} \sqrt{g^2 + g'^2}. \end{aligned} \quad (3.18)$$

for which the rotation matrix introduced in (2.11) takes the form

$$\begin{aligned}
 U_{Z'-\gamma \text{ kinetic and } Z'-Z \text{ mixing}} & \quad (3.19) \\
 &= \begin{pmatrix} 1 & 0 & -\frac{\sin \bar{\chi}}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} \\ 0 & 1 & -\frac{\sin \chi}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} \\ 0 & 0 & \frac{1}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} \end{pmatrix} \times U_{\text{Minimal } Z'-Z \text{ mass mixing}} \\
 &= \begin{pmatrix} \frac{g \cos \theta'}{\sqrt{g^2+g'^2}} + \frac{\sin \theta' \sin \bar{\chi}}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} & \frac{g'}{\sqrt{g^2+g'^2}} & \frac{g \sin \theta'}{\sqrt{g^2+g'^2}} - \frac{\cos \theta' \sin \bar{\chi}}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} \\ -\frac{g' \cos \theta'}{\sqrt{g^2+g'^2}} + \frac{\sin \theta' \sin \chi}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} & \frac{g}{\sqrt{g^2+g'^2}} & -\frac{g' \sin \theta'}{\sqrt{g^2+g'^2}} - \frac{\cos \theta' \sin \chi}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} \\ -\frac{\sin \theta'}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} & 0 & \frac{\cos \theta'}{\sqrt{1-\sin^2 \chi - \sin^2 \bar{\chi}}} \end{pmatrix}.
 \end{aligned}$$

5. Stueckelberg-type mixing [32–34]:

This kind of model provides mixing through the nonzero coupling constant \tilde{g}' and except for gauge coupling g, g', g'' , a typical choice as given in refs. [32, 33] is the vanishing of all other parameters

$$\beta_1 = \beta_2 = \beta_3 = \alpha_1 = \alpha_8 = \alpha_{24} = \alpha_{25} = 0, \quad (3.20)$$

leading to diagonal kinetic terms and mixing occurring only in the mass terms. After rotating the standard electroweak mixing angle θ_W , we can redefine the gauge fields

$$\begin{aligned}
 \bar{B}^\mu &= -\frac{g'' \sqrt{g^2 + g'^2}}{(g^2 + g'^2)g''^2 + g^2 \tilde{g}'^2} B_0^\mu + \frac{g \tilde{g}'}{(g^2 + g'^2)g''^2 + g^2 \tilde{g}'^2} Z_0'^\mu \\
 \bar{Z}'^\mu &= \frac{g \tilde{g}'}{(g^2 + g'^2)g''^2 + g^2 \tilde{g}'^2} B_0^\mu + \frac{g'' \sqrt{g^2 + g'^2}}{(g^2 + g'^2)g''^2 + g^2 \tilde{g}'^2} Z_0'^\mu
 \end{aligned} \quad (3.21)$$

thereby changing the present model to a minimal $Z' - Z$ mass mixing model with

$$\begin{aligned}
 M_{Z_0}^2 &= \frac{f^2}{4} (g^2 + g'^2 + \frac{4g'^2 \tilde{g}'^2}{g^2 + g'^2}) \\
 M_{Z_0'}^2 &= f^2 (g''^2 + \frac{g^2 \tilde{g}'^2}{g^2 + g'^2}) \\
 M_{Z Z'} &= -\frac{f^2 g' \tilde{g}' \sqrt{(g^2 + g'^2)g''^2 + g^2 \tilde{g}'^2}}{g^2 + g'^2}.
 \end{aligned} \quad (3.22)$$

The overall rotation matrix then becomes

$$\begin{aligned}
 U_{\text{Stueckelberg type mixing}} &= \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\sin \theta_W & \cos \theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{g'' \sqrt{g^2+g'^2}}{(g^2+g'^2)g''^2+g^2\tilde{g}'^2} & \frac{g \tilde{g}'}{(g^2+g'^2)g''^2+g^2\tilde{g}'^2} \\ 0 & \frac{g \tilde{g}'}{(g^2+g'^2)g''^2+g^2\tilde{g}'^2} & \frac{g'' \sqrt{g^2+g'^2}}{(g^2+g'^2)g''^2+g^2\tilde{g}'^2} \end{pmatrix} \\
 &\times \begin{pmatrix} \cos \theta' & 0 & \sin \theta' \\ 0 & 1 & 0 \\ -\sin \theta' & 0 & \cos \theta' \end{pmatrix}
 \end{aligned} \quad (3.23)$$

with θ' evaluated from the second equation of (3.6) and those of (3.22). In ref. [34], the Stueckelberg-type mixing is further generalized to include kinetic mixing by relaxing the original condition $\alpha_{25} = 0$. This kinetic mixing can be diagonalized by applying (3.10) and following a similar procedure to that leading to (3.21) in diagonalizing the mass terms.

4 The Z' boson charges to quark and leptons

The charges for the Z' boson with respect to ordinary quarks and leptons can be expressed in terms of the gauge interaction as

$$\mathcal{L}_{\text{gauge coupling}} = g'' X_\mu J_X^\mu \quad J_X^\mu = \sum_i \bar{f}_i \gamma^\mu [y'_{iL} P_L + y'_{iR} P_R] f_i, \quad (4.1)$$

where index i distinguishes the three generations associated with the six quarks u, c, t, d, s, b and six leptons $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$, and y'_{iL}, y'_{iR} are the corresponding left- and right-hand charges.⁴ The $SU(2)_L$ symmetry requires equating $U(1)'$ charges of the two components of the left-hand fermion doublet, i.e. $y'_{u,L} = y'_{d,L} \equiv y'_q$ for quark and $y'_{\nu,L} = y'_{e,L} \equiv y'_l$ for lepton. Thus, we can parameterize the fermionic $U(1)'$ charges by $y'_q, y'_u, y'_d, y'_l, y'_e$ and y'_ν . In general, the assignments of $U(1)'$ charges are generation-dependent, but in its simplest form $U(1)'$ charges can be generation-independent, much like hypercharge assignments in SM. TABLE. 1 lists four sets of more common assignments for the generation-independent $U(1)'$ charges of fermions in new physics models involving Z' boson [21, 35]. In the $U(1)_{B-xL}$ model (see column 3 of TABLE. 1), Z' charges are determined by the baryon number and lepton number from $y'_i = B_i - xL_i$ with a free rational parameter x . Leptophobic and hadrophobic Z' models correspond to $x = \infty$ and $x = 0$, respectively. The second set of charges comes from grand unified theories. Parameter x establishes the mixing of the two extra $U(1)$ groups in the E_6 symmetry breaking patterns $E_6 \rightarrow SU(5) \times U(1) \times U(1)$. Z_χ, Z_ψ and Z_η of ref. [36] correspond to the special case with $x = -3, x = 1$ and $x = -1/2$, respectively. The third set, $U(1)_{d-xu}$ results in the vanishing of the left-hand quark doublet charge and the ratio of right-hand up quark charges to down quark charges is controlled by $-x$. In the last set, the free parameter x is the ratio of the charges of the left-hand quark doublet and right-hand up quark singlet and reduces to the $U(1)_{B-L}$ model for $x = 1$. Theoretically, the charges of quarks and leptons must satisfy the anomaly cancellation conditions to preserve the gauge symmetry. We now examine the constraints on generation-independent $U(1)'$ charges arising as a consequence of these anomaly cancellation conditions. Davidson et.al. [37] have studied anomaly cancellation for additional $U(1)'$ gauge group and derived the following anomaly cancellation conditions for $U(1)_Y \otimes U(1)'$ gauge groups

$$\sum y_L^\alpha = \sum Q^2 (y_L^\alpha - y_R^\alpha) = 0 \quad \sum Q (y_L^\alpha y_L^\beta - y_R^\alpha y_R^\beta) = 0 \quad \sum (y_L^\alpha y_L^\beta y_L^\gamma - y_R^\alpha y_R^\beta y_R^\gamma) = 0, \quad (4.2)$$

⁴Phenomenologically, we need to further express the gauge interaction given in eq. (4.1) in terms of mass eigenstate of Z' , for then the $Z' - Z - \gamma$ mixings discussed in the last section set in.

models	Z' EWCL	$U(1)_{B-xL}$	$U(1)_{10+x\bar{5}}$	$U(1)_{d-xu}$	$U(1)_{q+xu}$
(u_L, d_L)	y'_q	$1/3$	$1/3$	0	$1/3$
u_R	y'_u	$1/3$	$-1/3$	$-x/3$	$x/3$
d_R	y'_d	$1/3$	$-x/3$	$1/3$	$(2-x)/3$
(ν_L, e_L)	y'_l	$-x$	$x/3$	$(x-1)/3$	-1
e_R	y'_e	$-x$	$-1/3$	$x/3$	$-(2+x)/3$
ν_R	y'_{ν_R}	-1	$(x-2)/3$	$-x/3$	$(x-4)/3$

Table 1. Generation-independent $U(1)'$ charges for quarks and leptons.

where α, β, γ indexes $U(1)_Y$ and $U(1)'$ charges. Substituting the $U(1)_Y$ charges for ordinary quarks and leptons and assuming the generation-independence of $U(1)'$ charges, we find that above equations imply

$$\begin{cases} y'_l + 3y'_q = 0 \\ 3y'_l + 5y'_q - 3y'_e - 4y'_u - y'_d = 0 \\ -y_l'^2 + y_q'^2 + y_e'^2 - 2y_u'^2 + y_d'^2 = 0 \\ 3y'_l + y'_q - 6y'_e - 8y'_u - 2y'_d = 0 \\ 2y_l'^3 + 6y_q'^3 - y_e'^3 - 3y_u'^3 - 3y_d'^3 - y_{\nu_R}'^3 = 0 \end{cases} . \quad (4.3)$$

The last equation in (4.3) can be satisfied by assigning y'_{ν_R} a proper value or adding in our theory some other new fermions. Solving the above equations, we obtain two sets of solutions which satisfy the anomaly cancellation conditions

$$\begin{cases} y'_l = -3y'_q \\ y'_d = 2y'_q - y'_u \\ y'_e = -2y'_q - y'_u \\ y'_{\nu_R} = -4y'_q + y'_u \end{cases} \quad \text{or} \quad \begin{cases} y'_l = -3y'_q \\ y'_d = -\frac{14}{5}y'_q + \frac{1}{5}y'_u \\ y'_e = -\frac{2}{5}y'_q - \frac{7}{5}y'_u \\ y'_{\nu_R} = \frac{\sqrt[3]{35}}{5}(4y'_q - y'_u) \end{cases} . \quad (4.4)$$

Of the six of $U(1)'$ charges, only two of them y'_q and y'_u are independent; the other four being linear combinations of these two. In addition, there are two kinds of linear combinations: the first of eq. (4.4) which was given and discussed in detail in ref. [19], while the second is a new solution having not yet appeared in the literature. We can utilize the values of y'_q and y'_u to classify the new physics models and in the following we list some typical cases:

1. Left Handed: $y'_u = y'_d = y'_e = y'_{\nu_R} = 0 \Rightarrow y'_q = y'_l = 0$
2. Right Handed: $y'_q = y'_l = 0 \Rightarrow y'_d = -y'_u = y'_e = -y'_{\nu_R}$ or $y'_d = \frac{1}{5}y'_u = -\frac{1}{7}y'_e = -\frac{1}{\sqrt[3]{35}}y'_{\nu_R}$
3. Left-Right symmetric: $y'_q = y'_u = y'_d \Rightarrow y'_l = y'_e = y'_{\nu_R} = -3y'_q$
4. ν_R decouple: $y'_{\nu_R} = 0 \Rightarrow y'_u = 4y'_q, y'_e = 2y'_l = 3y'_d = -6y'_q$

Checking the assignments given in TABLE. 1 against the two solutions in (4.4), we find that the $U(1)_{B-xL}$, $U(1)_{d-xu}$ and $U(1)_{q+xu}$ models are anomaly-free when parameter $x = 1$ with the right-hand neutrino charge $y'_{\nu_R} = -1$, $y'_{\nu_R} = -\frac{1}{3}$ and $y'_{\nu_R} = -1$, respectively.

models	$U(1)_{B-xL_e-yL_\mu}$	$U(1)_{10+x\bar{5}}$ gen-dep	$U(1)_{d-xu}$ gen-dep	$U(1)_{q+xu+yc+zt}$	2+1 leptocratic
$q_{1,L}$	1/3	1/3	0	1/3	1/3
u_R	1/3	-1/3	$-x/3$	$x/3$	$x/3$
d_R	1/3	$-x/3$	1/3	$(2-x)/3$	$(2-x)/3$
$q_{2,L}$	1/3	1/3	0	1/3	1/3
c_R	1/3	-1/3	$-y/3$	$y/3$	$x/3$
s_R	1/3	$-y/3$	1/3	$(2-y)/3$	$(2-x)/3$
$q_{3,L}$	1/3	1/3	0	1/3	1/3
t_R	1/3	-1/3	$2 - \frac{2}{3}(x+y) \pm \sqrt{3-x^2-y^2}$	$z/3$	$x/3$
b_R	1/3	$3 + \frac{x+y}{3}$	1/3	$(2-z)/3$	$(2-x)/3$
(ν_L^e, e_L)	$-x$	$x/3$	$(x-1)/3$	-1	$-1-2y$
e_R	$-x$	-1/3	$x/3$	$-(2+x)/3$	$-(2+x)/3-2y$
(ν_L^μ, μ_L)	$-y$	$y/3$	$(y-1)/3$	-1	$y-1$
μ_R	$-y$	-1/3	$y/3$	$-(2+y)/3$	$-(2+x)/3+y$
(ν_L^τ, τ_L)	$x+y-3$	$3 + \frac{x+y}{3}$	$\frac{2}{3} - \frac{1}{3}(x+y)$	-1	$y-1$
τ_R	$x+y-3$	-1/3	$x+y-3 \mp \frac{4}{3}\sqrt{3-x^2-y^2}$	$-(2+z)/3$	$-(2+x)/3+y$

Table 2. Generation-dependent charge.

Furthermore, the $U(1)_{10+x\bar{5}}$ model is anomaly-free when $x = -3$ with $y'_{\nu_R} = -5/3$. Even though the anomaly cancellation condition can not be satisfied with the present quarks and leptons, we still have the possibility of canceling the anomalies by adding some extra fermions into theory.

If we relax the generation-independence criterion on the $U(1)'$ charges, we need to add generation indices to each of the charges in eq. (4.3) and sum over the generations on the left-hand side of eq. (4.3). In this case, there are too many free parameters and solutions. We list several possible solutions in TABLE. 2, in which the first and last columns are the two solutions given in ref. [35], and the remaining solutions can be seen to be some kind of generation-dependent generalization of charge assignments given in the third, fourth and fifth columns in TABLE. 1. The typical feature of these solutions is that for the solutions given in the first four columns of TABLE. 2, the charges for the first two generations are parameterized in a like manner as those in the generation-independent situation by x or y separately, and differences appear only in the third generation of quarks and leptons. Of special note is that for the solution to $U(1)_{q+xu+yc+zt}$, the anomaly cancellation condition is satisfied for each generation independently.

5 Summary

In this paper, we have classified various new physics models involving the Z' boson in two different ways: one according to Z' boson mixings with Z and γ , and the other according to Z' boson charges with respect to quarks and leptons. In regard to the former, we based the general description for the $Z'-Z-\gamma$ mixing derived from the EWCL on our previous work [8], characterizing these new physics models into five classes: 1. Models with minimal $Z'-Z$ mass mixing; 2. Models with minimal $Z'-Z$ kinetic mixing; 3. Models with general $Z'-Z$ mixing; 4. Models with $Z'-\gamma$ kinetic and $Z'-Z$ mixing; and 5. Models with Stueckelberg-type mixing. Although the general $Z'-Z-\gamma$ mixing is complicated and there

is no exact analytical expression for the mixing matrix U and masses $M_Z, M_{Z'}$, we obtain explicit analytical expressions for each of our five simplifying classes. We find that the most elementary mixing is the minimal $Z' - Z$ mass mixing, the other four classes of mixings can be transformed into the minimal $Z' - Z$ mass mixing through field transformations. In regard to the latter classification, we exploit the anomaly cancellation conditions to constrain the $U(1)'$ charges. For generation-independent $U(1)'$ charges, there are six charges $y'_q, y'_u, y'_d, y'_l, y'_e, y'_\nu$ for which anomaly cancellation requires that only two are independent parameters while the other four can depend on these two parameters in two different ways. While one appears already in the literature, the other is new. For generation-dependent $U(1)'$ charges, we have listed some possible special solutions.

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